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# THE UNIVERSITY OF MICHIGAN

## COLLEGE OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Radiation Laboratory

ANTENNA ARRAY RESEARCH

Final Technical Report

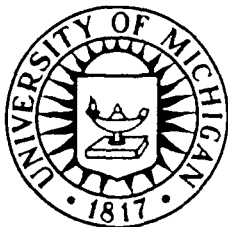
Grant NGR-23-005-477

July 1973

(1 July 1972 - 30 June 1973)

By:  
Chen-To Tai

**CASE FILE  
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This is the final report summarizing the work done on NASA Grant 23-005-477 during the period from July 1, 1972 through June 30, 1973.

Three topics have been covered in our study. The results are summarized here under separate headings.

## 1 - On Electromagnetic Field Problems in Inhomogeneous Media

This study deals with a general theoretical investigation of the field representation in an inhomogeneous media. It is an extension of the earlier work on linearly stratified media and spherically stratified media [ 1 ]. We consider here stratification in a general curvilinear orthogonal coordinate system. A technical report [ 2 ], describing a general method of solving the differential equation resulting from such a formulation, was prepared and submitted to the NASA Scientific and Technical Information Office in January, 1973. The work has been submitted to a technical journal for consideration of publication.

## 2 - Comparison of the Radiation Pattern and Directivity of Small Luneburg Lens Versus Homogeneous Lens

It has been assumed that a spherical Luneburg lens, because of its focusing property from the point of view of geometrical theory of diffraction, could be a better antenna than a homogeneous spherical dielectric lens for lens diameters not too small. In order to test this assertion a detailed calculation has been made for the Luneburg lens based on the exact formulation [ 3 ]. Both the ordinary and the generalized confluent hypergeometric functions encountered in the theory of Luneburg lens

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have been computed for different orders as a function of  $2\pi a/\lambda$  where 'a' denotes the radius of the lens. Several typical radiation patterns are shown in Figures 1 through 8 for lens excited by a Huygen's source. In the same figures we have plotted the patterns for two homogeneous lenses with  $\epsilon_r$ , respectively, equal to 1.667 and 3.00. Figure 9 plots the directivity of these lenses for  $2a/\lambda \leq 5$ . It is seen from these plots that a moderately sized Luneburg lens does not exhibit a better characteristic than a homogeneous lens with  $\epsilon_r = 3.00$ , although it is superior than a homogeneous lens with  $\epsilon_r = 1.667$ . From this work it is concluded that there is no advantage to replace a homogeneous lens by a Luneburg lens as the feeds for waveguides [4]. A homogeneous lens with an optimum value of  $\epsilon_r$  in the neighborhood of 2.57 appears to be the best feed for such a system [5].

## 3 - Characteristic of Eaton Lens

For certain applications, one is interested to have antenna feeds with an omnidirectional pattern, at least for hemispherical coverage, in contrast to the directional feeds described in the previous section. It is known from the geometrical theory of diffraction that an Eaton lens with a dielectric distribution of the form

$$\epsilon_r = \left( \frac{R}{a} \right)^2 \quad a \geq R > 0$$

would produce a cylindrical phase front around the lens. It is, therefore, anticipated that such a lens may provide an omnidirectional pattern when it is excited by a Huygen's source. In the electromagnetic theory of such a lens, the key functions to be computed involve Bessel functions of fractional

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order. Several radiation patterns for an Eaton Lens fed by a Huygen's source have been calculated. They are shown in Figures 10 through 13. For lens with a radius smaller than one wavelength ( $a < \lambda$ ), the radiation pattern does not exhibit a large variation. For larger lenses, it appears that the pattern tends to be more directive in the broadside direction ( $\theta = \pi / 2$ ). Although our study is not yet conclusive, it seems that for certain sizes of the lens,  $a = 1.06\lambda$  for example, one can indeed obtain a fairly omnidirectional pattern in both planes. Further work needs to be done to ascertain the detailed characteristics of the Eaton lens.

Several members of the Radiation Laboratory have contributed to the work reported here. Included are Professor C-T Tai, Dr. Adel Mohsen, a post-doctoral fellow of the laboratory and Mr. P. Rozenfeld, a graduate student of the Department of Electrical and Computer Engineering of the University of Michigan. The support of this work by the National Aeronautics and Space Administration is duly acknowledged. The technical guidance provided by Mr. William F. Croswell of the Langley Research Center has been most helpful in the course of this study.

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## References

- [ 1 ] Tai, Chen-To, Dyadic Green's Functions in Electromagnetic Theory, Intext Educational Publishers, Scranton, Pa. 1971, Chapter 12.
- [ 2 ] Mohsen, Adel, "On Electromagnetic Field Problems in Inhomogeneous Media", Technical Report 010106-2-T, Radiation Laboratory, The University of Michigan, (January 1973), prepared for NASA Langley Research Center under Grant NGR 23-005-477.
- [ 3 ] Tai, Chen-To, loc. cit. Chapter 12.
- [ 4 ] Croswell, William F and J.S.Chatterjee, "Waveguide Excited Dielectric Sphere as Feeds", Trans. IEEE, AP-20, p. 206-208, 1972.
- [ 5 ] Mason, V. Bradford, "The Electromagnetic Radiation From Simple Sources in the Presence of a Homogeneous Dielectric Sphere", Ph.D. Dissertation, Department of Electrical and Computer Engineering, The University of Michigan, Ann Arbor, Michigan (1972).

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## Figure Captions

1. Radiation pattern of Luneburg lens and homogeneous lens excited by a Huygen's source; E-plane,  $D = 4.23 \lambda$ .
2. Radiation pattern of Luneburg lens and homogeneous lens excited by a Huygen's source; H-plane,  $D = 4.23 \lambda$ .
3. Radiation pattern of Luneburg lens and homogeneous lens excited by a Huygen's source; E-plane,  $D = 3.39 \lambda$ .
4. Radiation pattern of Luneburg lens and homogeneous lens excited by a Huygen's source, H-plane,  $D = 3.39 \lambda$ .
5. Radiation pattern of Luneburg lens and homogeneous lens excited by a Huygen's source; E-plane,  $D = 2.12 \lambda$ .
6. Radiation pattern of Luneburg lens and homogeneous lens excited by a Huygen's source; H-plane,  $D = 2.12 \lambda$ .
7. Radiation pattern of a Luneburg lens and homogeneous lens excited by a Huygen's source; E-plane,  $D = 1.27 \lambda$ .
8. Radiation pattern of Luneburg lens and homogeneous lens excited by a Huygen's source; H-plane,  $D = 1.27 \lambda$ .
9. Directivity of Luneburg lens and homogeneous lens.
10. Radiation pattern of Eaton lens excited by a Huygen source,  $D = 4.23 \lambda$ .
11. Radiation pattern of Eaton lens excited by a Huygen source,  $D = 3.39 \lambda$ .
12. Radiation pattern of Eaton lens excited by a Huygen source,  $D = 2.12 \lambda$ .
13. Radiation pattern of Eaton lens excited by a Huygen source,  $D = 1.27 \lambda$ .

210°  
150°200°  
160°190°  
170°

180°

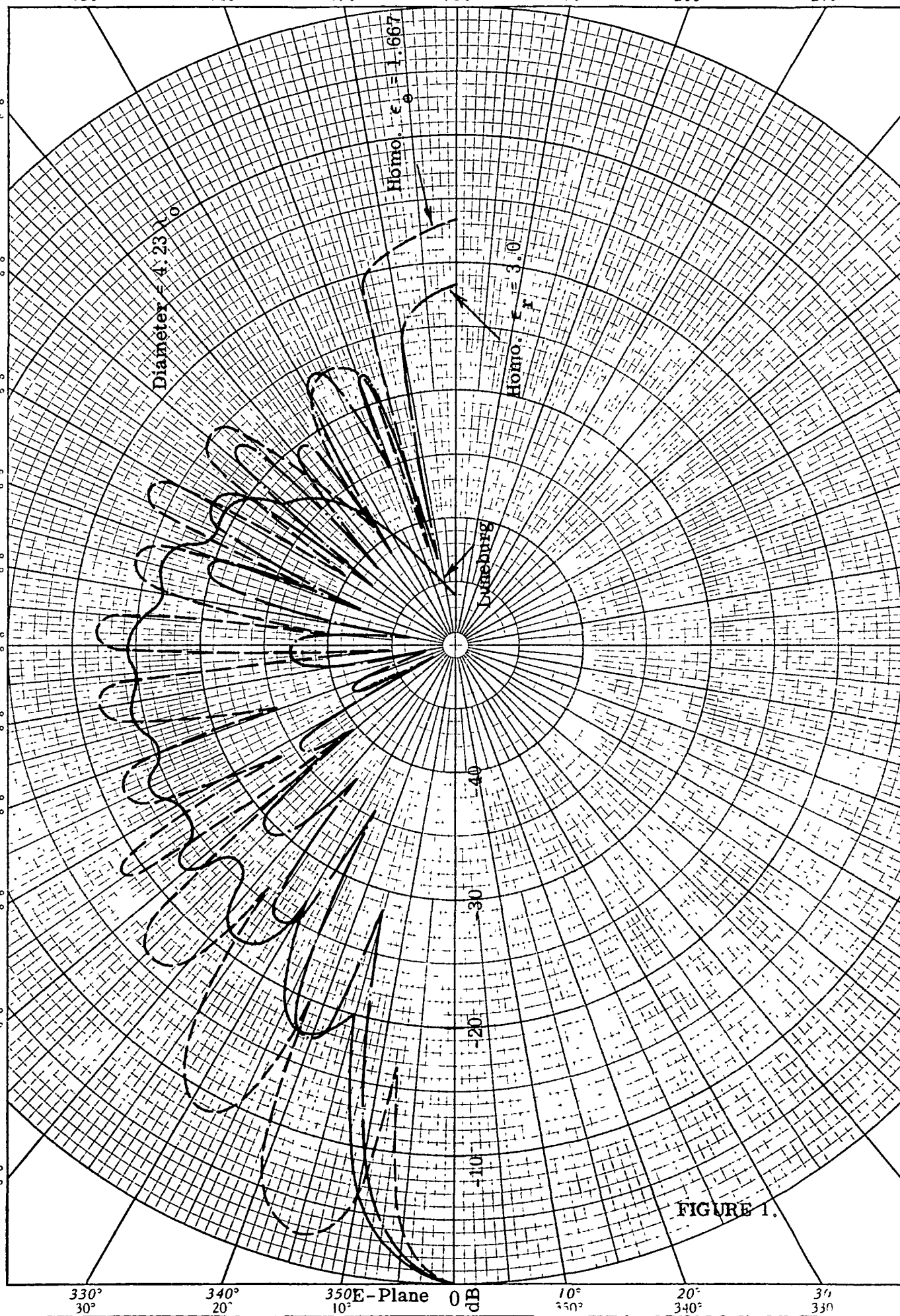
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200°150°  
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80°290°  
70°300°  
60°310°  
50°320°  
40°140°  
220°130°  
230°120°  
240°110°  
250°100°  
260°90°  
270°80°  
280°70°  
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320°

FIGURE 1.

210°  
150°200°  
160°190°  
170°

180°

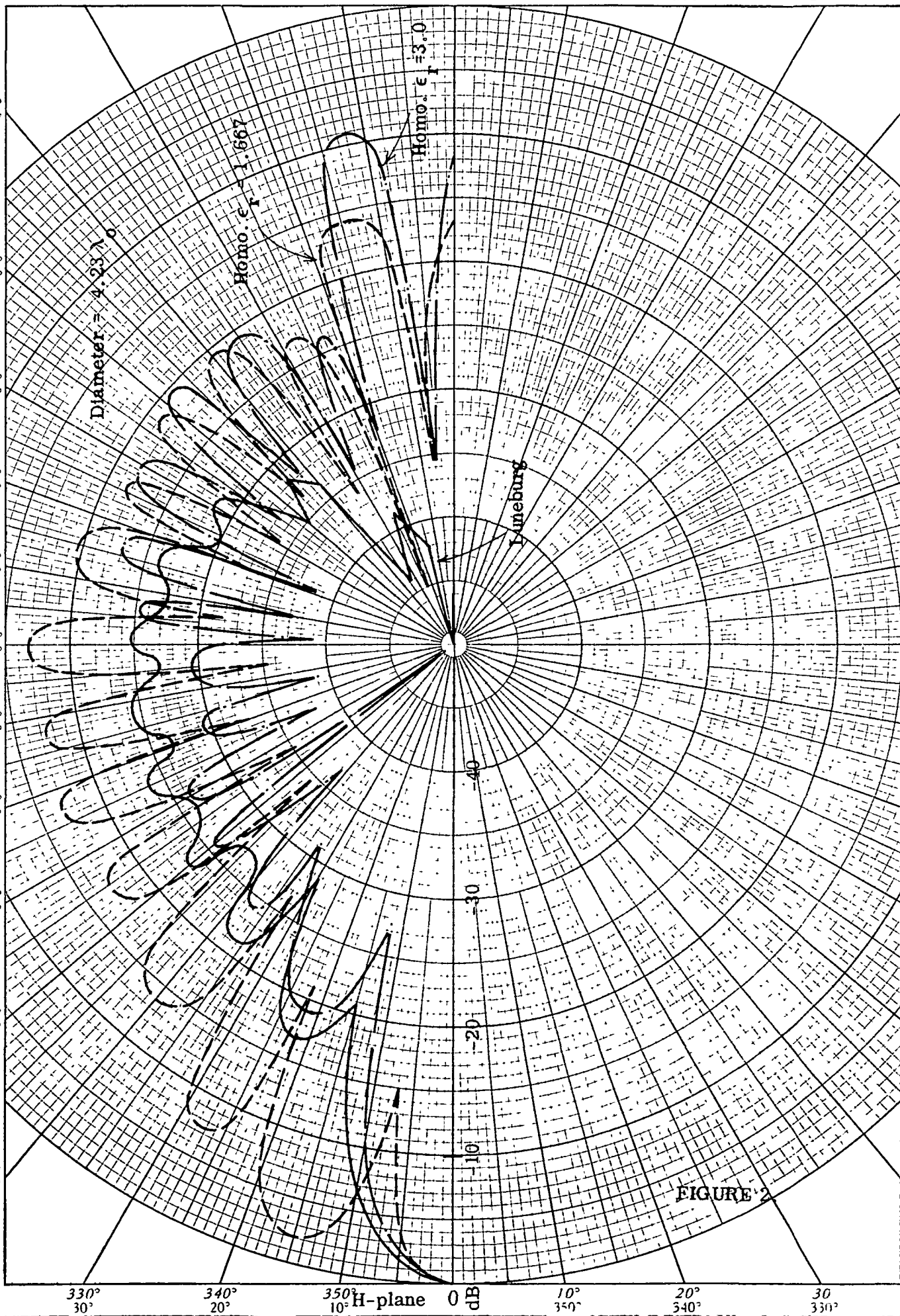
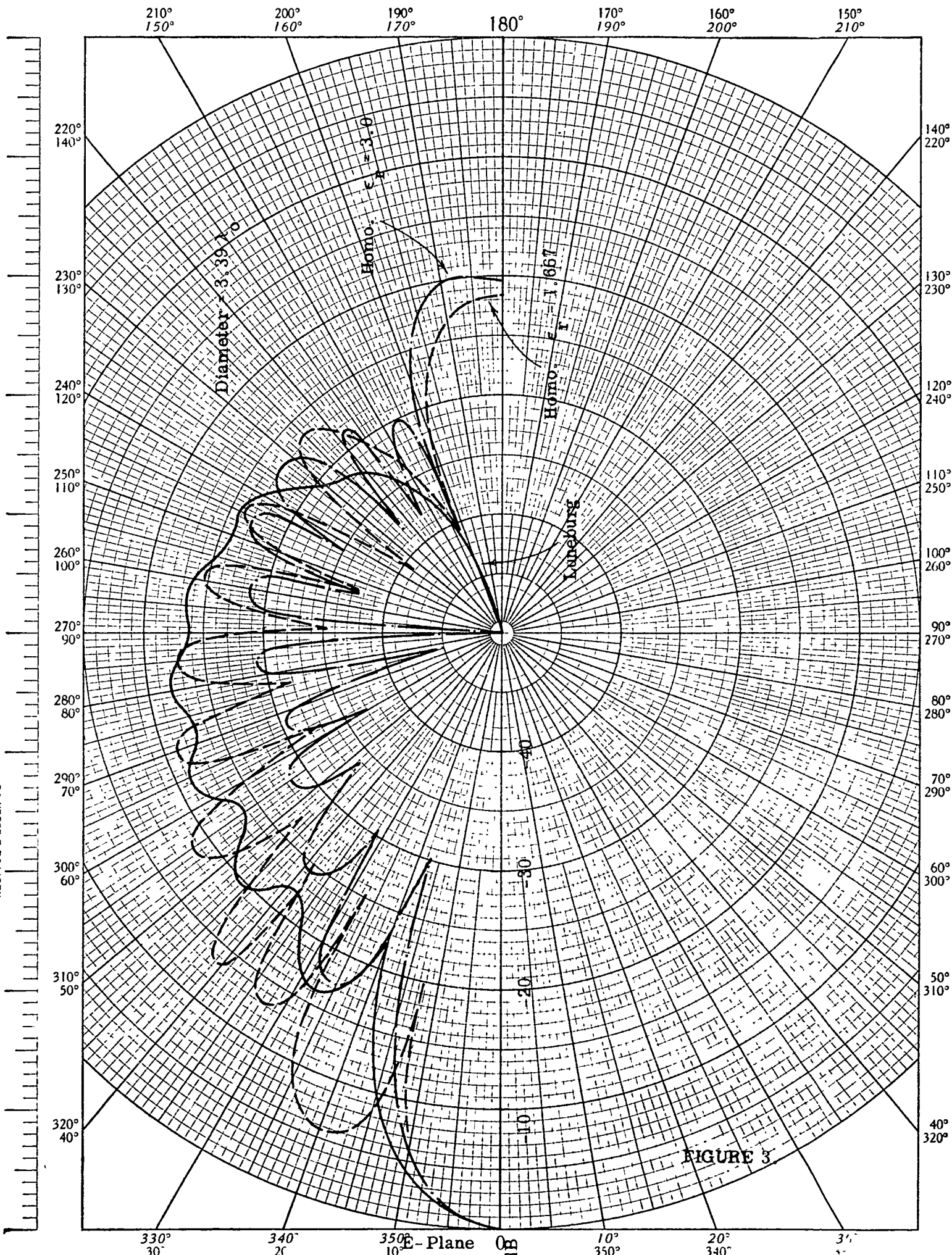
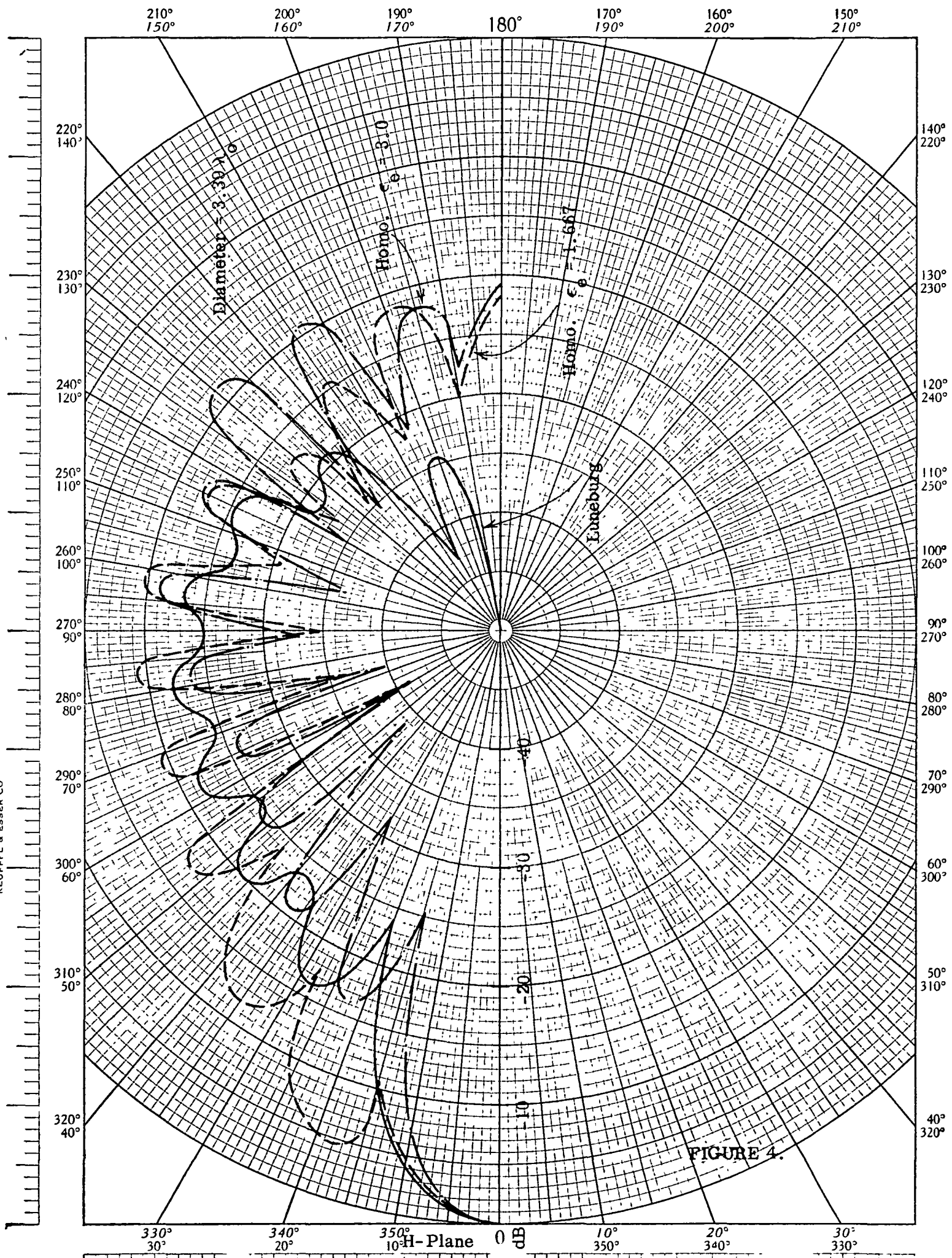
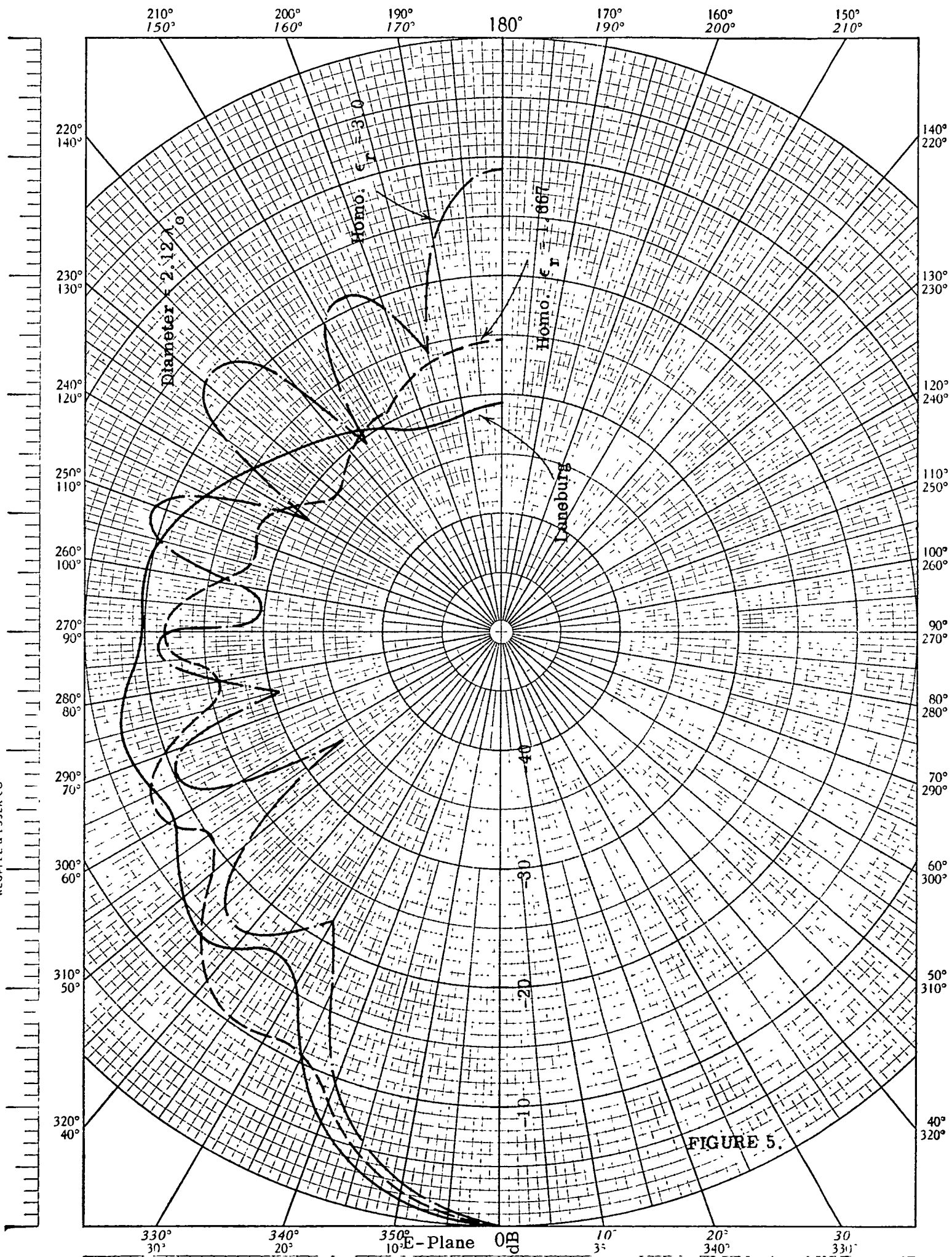
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210°220°  
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50°320°  
40°140°  
220°130°  
230°120°  
240°110°  
250°100°  
260°90°  
270°80°  
280°70°  
290°60°  
300°50°  
310°40°  
320°

FIGURE 2









210°  
150°200°  
160°190°  
170°

180°

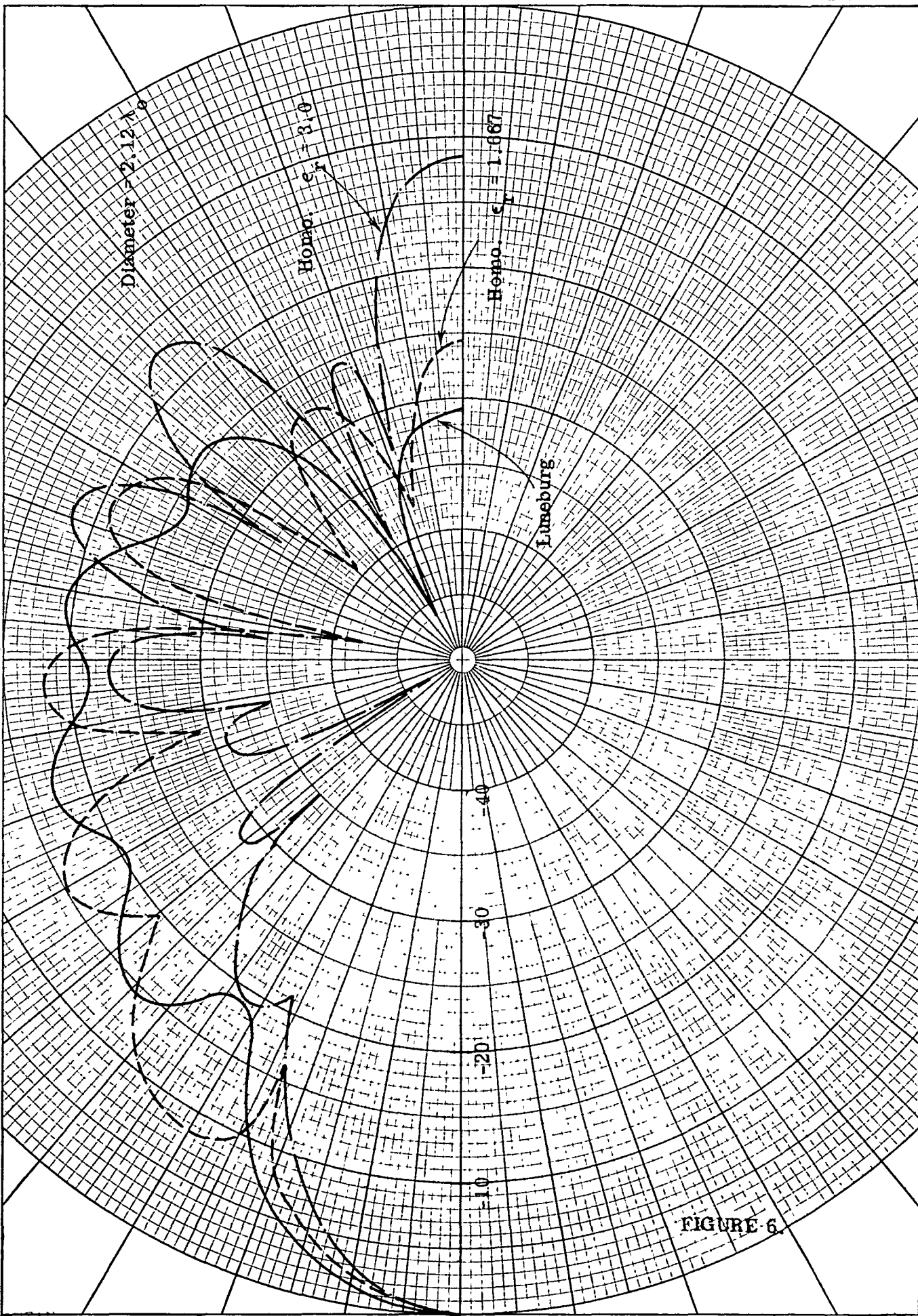
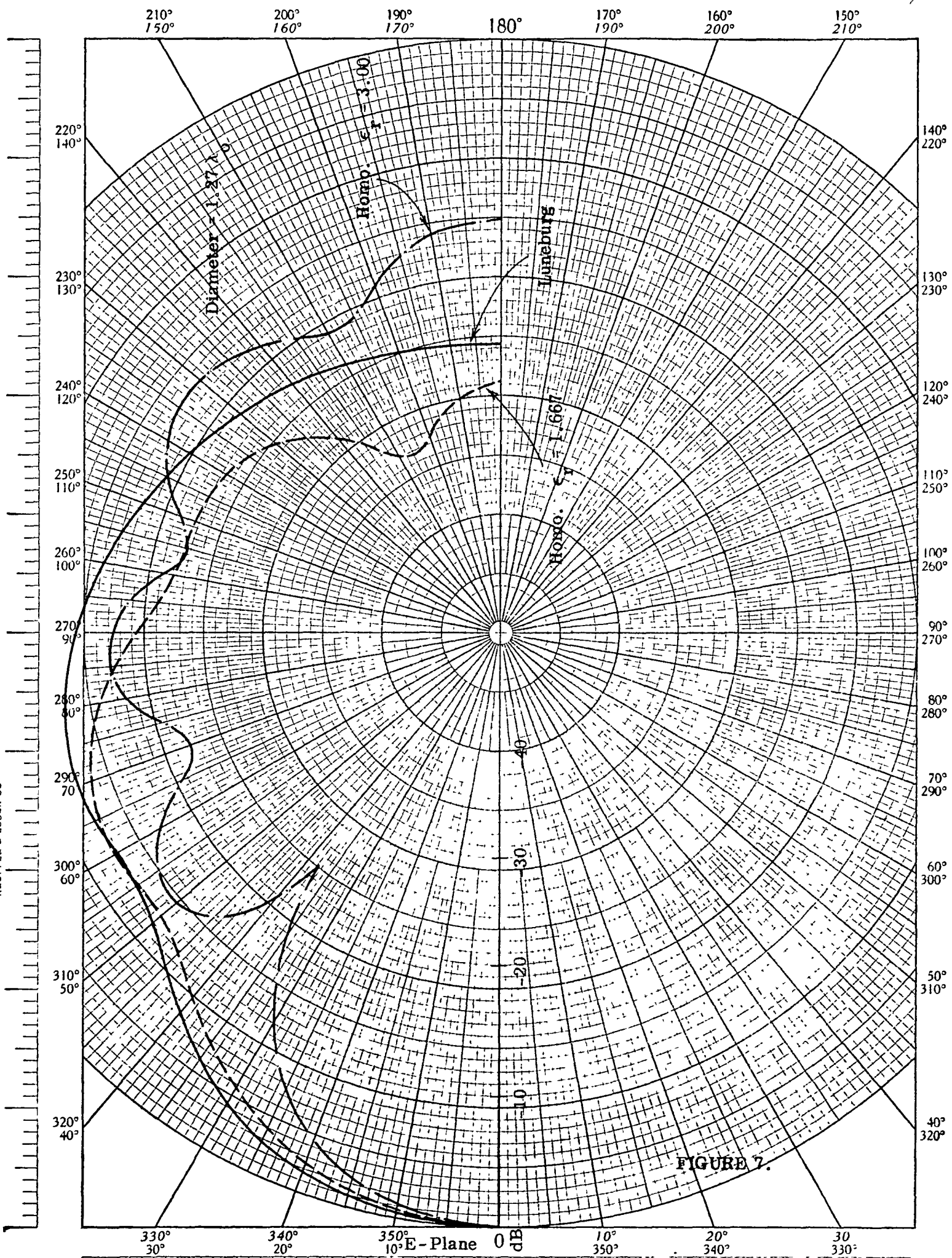
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FIGURE 6.





210°  
150°200°  
160°190°  
170°

180°

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190°160°  
200°150°  
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240°110°  
250°100°  
260°90°  
270°80°  
280°70°  
290°60°  
300°50°  
310°40°  
320°Diameter = 1.271<sub>0</sub>Horn.  $\epsilon_r = 3.0$ 

Luneburg

Horn.  $\epsilon_r = 1.667$ 

10° H-Plane 0 dB

10°  
350°20°  
340°30°  
330°

FIGURE 8.

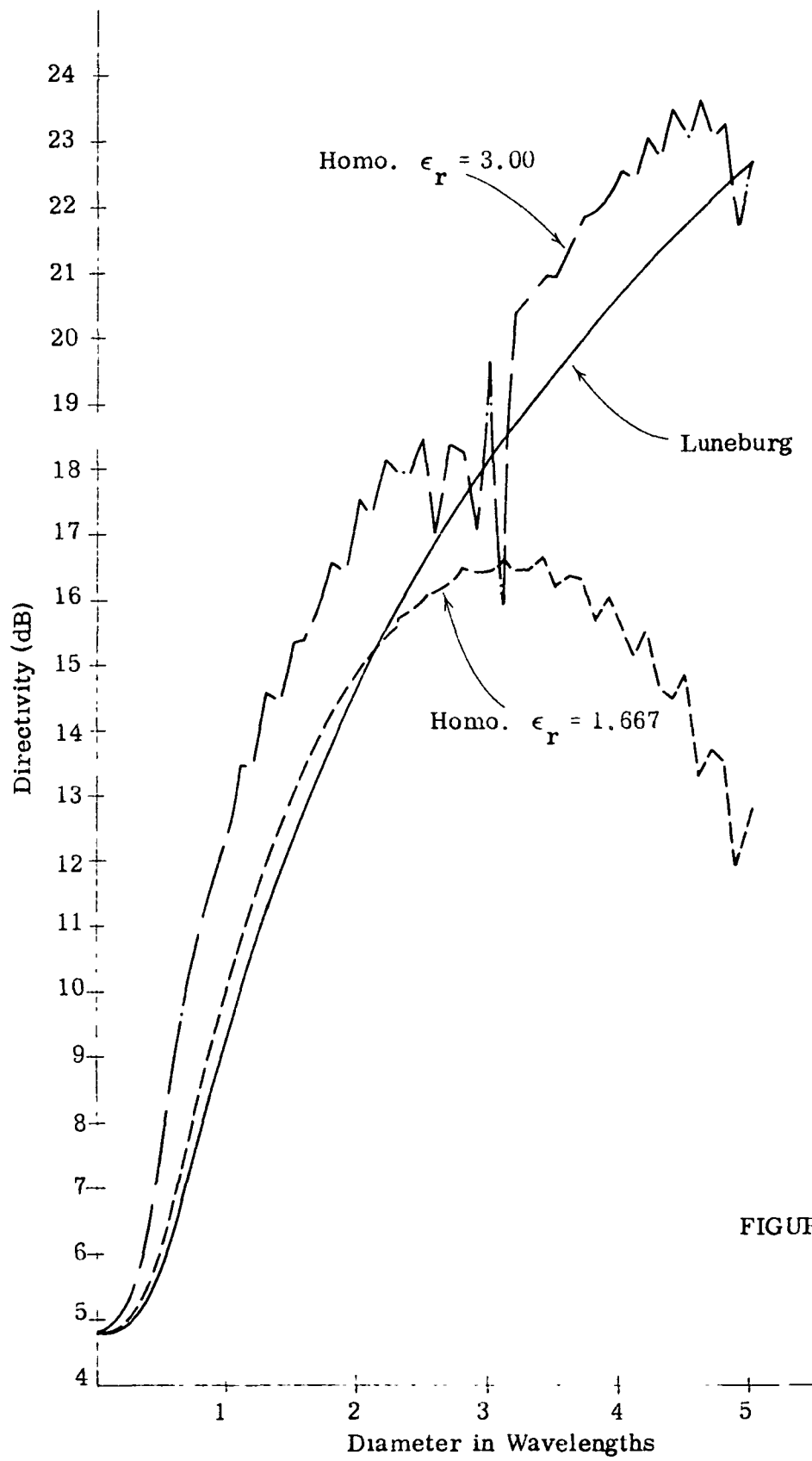


FIGURE 9.

210°  
150° E-Plane200°  
160°190°  
170°

180°

170°  
190°160°  
200° H-Plane150°  
210°220°  
140°230°  
130°240°  
120°250°  
110°260°  
100°270°  
90°280°  
80°290°  
70°300°  
60°310°  
50°320°  
40°330°  
30°340°  
20°350°  
10°

0

350°

340°

330°

320°

310°

300°

290°

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260°

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230°

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10°

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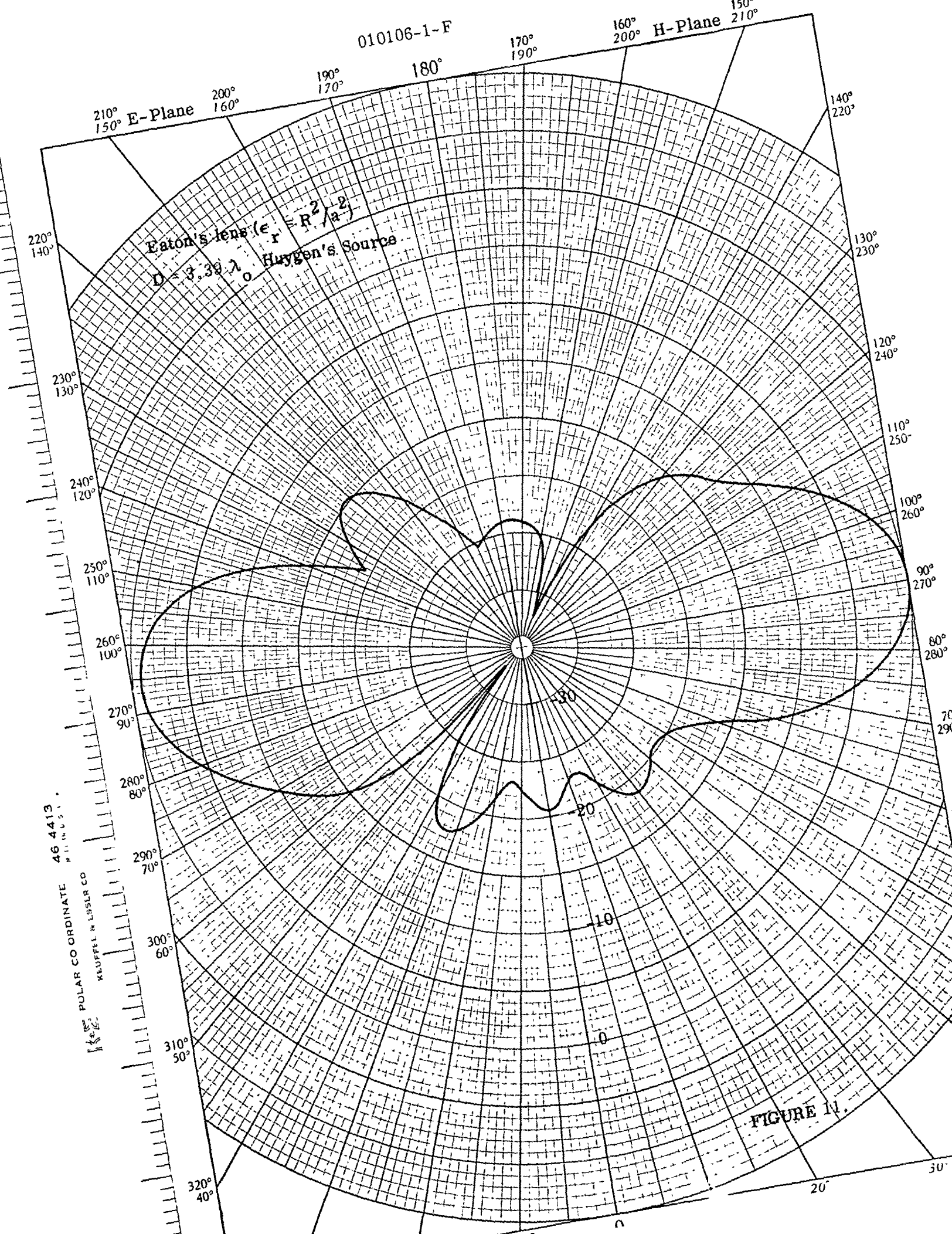
220°

210°

200°

190°





210°  
150° E-Plane200°  
160°190°  
170°

180°

170°  
190°160°  
200° H-Plane150°  
210°

Eaton's lens ( $\epsilon_r = R^2/a^2$ )  
 $D = 2.12 \lambda$  Huygen's Source

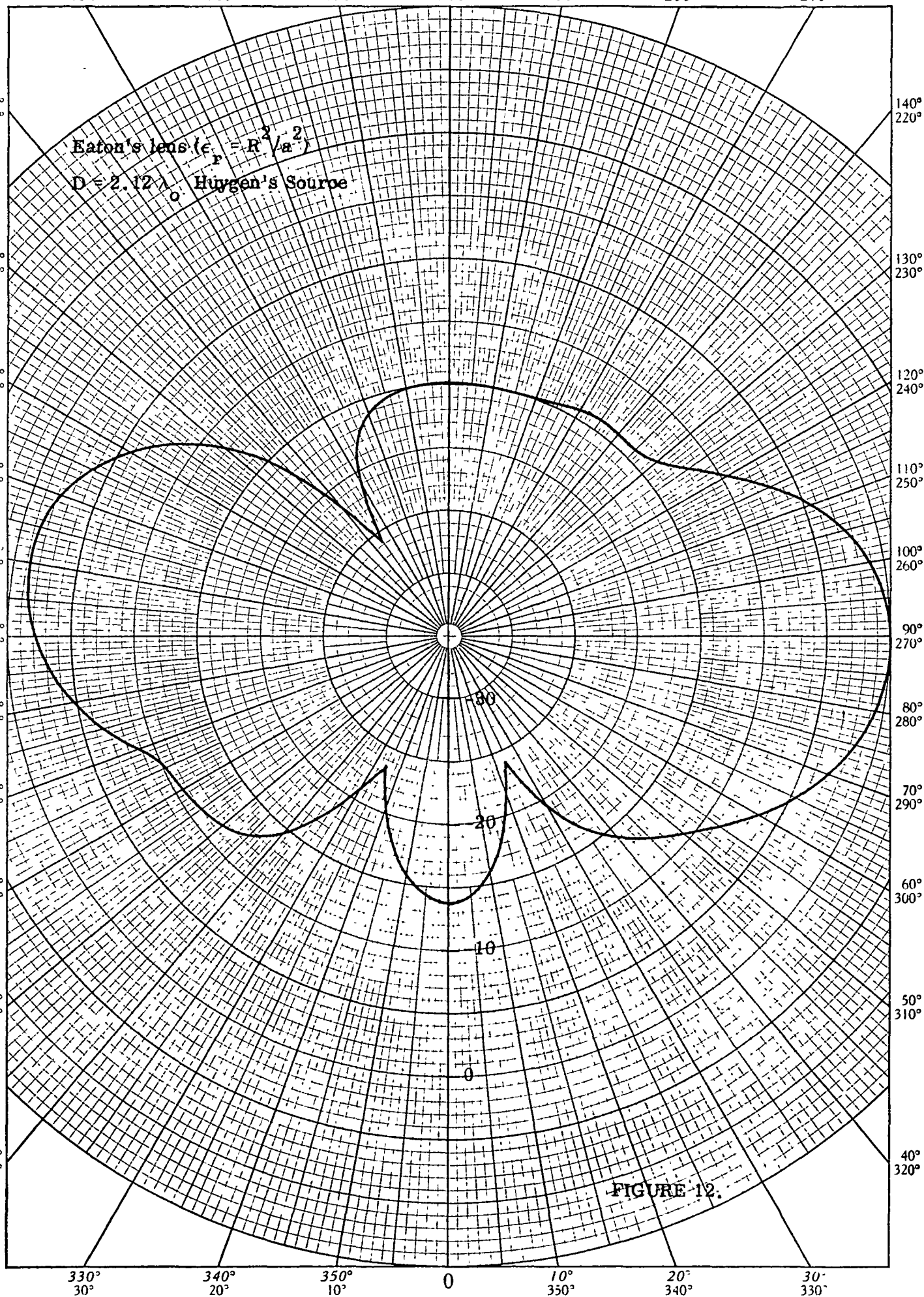


FIGURE 12.

210°  
150° E-Plane200°  
160°190°  
170°

180°

170°  
190°160°  
200° H-Plane150°  
210°

Eaton's lens ( $\epsilon_r = R^2/a^2$ )  
 $D = 1.27 \lambda_0$  Huygen's Source

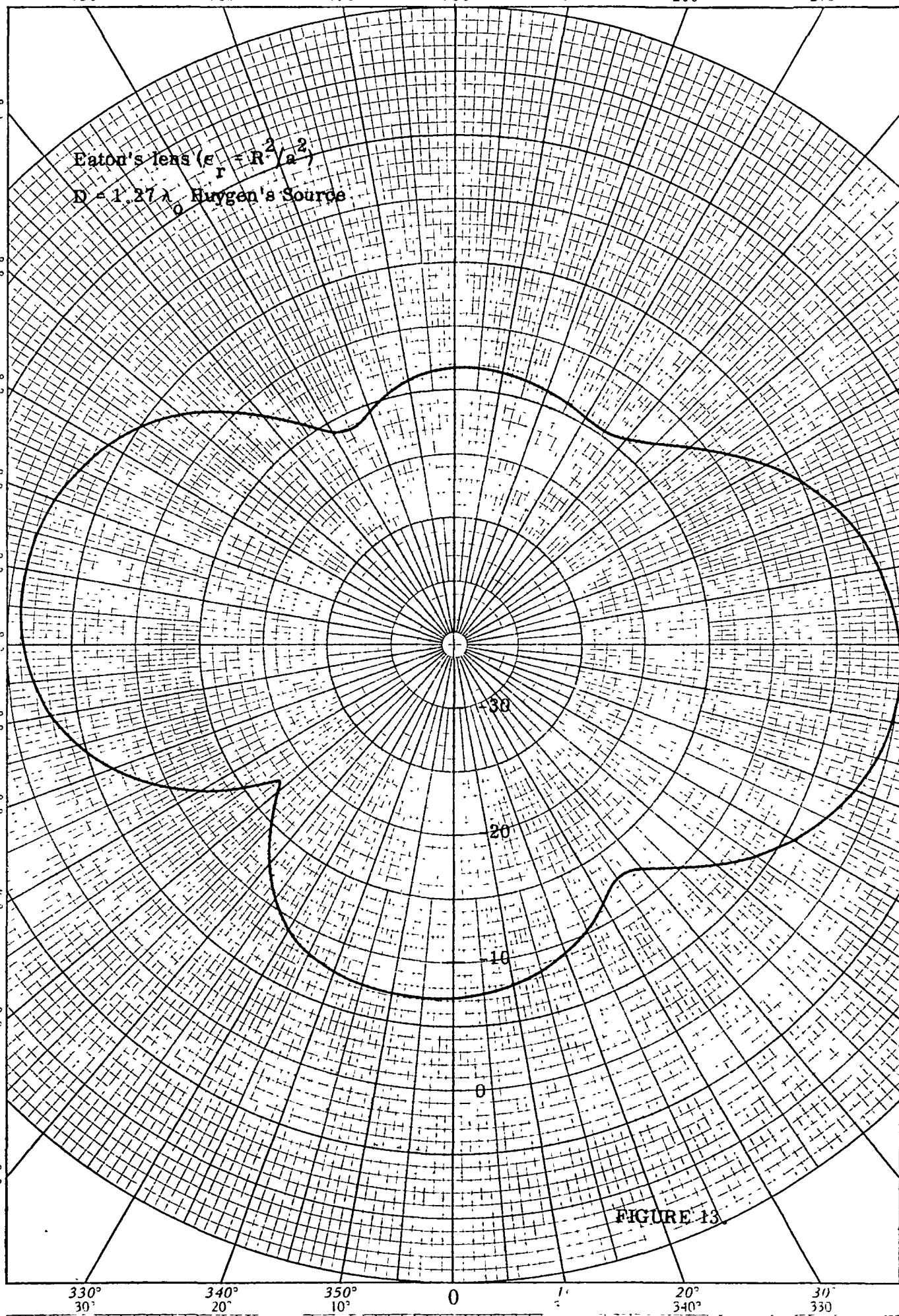


FIGURE 13